Roll No : ....

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ASP–648

M.A./M.Sc. (Final) Examination, 2021

MATHEMATICS

Paper - Opt. VI (Due Paper-VIII)

(Topology)

Time : 1½ Hours]

[*Maximum Marks : **100***

Section–A **(Marks : 2 × 10 = 20)**

Note :– Answer all *ten* questions (Answer limit **50** words). Each question carries **2** marks.

(खण्ड–अ) **(अंक : 2 × 10 = 20)**

नोट :– सभी **दस** प्रश्नों के उत्तर दीजिए (उत्तर-सीमा **50** शब्द)। प्रत्येक प्रश्न **2** अंक का है।

Section–B **(Marks : 4 × 5 = 20)**

Note :– Answer all *five* questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

(खण्ड–ब) **(अंक : 4 × 5 = 20)**

नोट :– सभी **पाँच** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न में विकल्प का चयन कीजिए (उत्तर-सीमा **200** शब्द)। प्रत्येक प्रश्न **4** अंक का है।

Section–C **(Marks : 20 × 3 = 60)**

Note :– Answer any *three* questions out of five (Answer limit **500** words). Each question carries **20** marks.

(खण्ड–स) **(अंक : 20 × 3 = 60)**

नोट :– पाँच में से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए (उत्तर-सीमा **500** शब्द)। प्रत्येक प्रश्न **20** अंक का है।

Section–A

1. Define the following :

   (i) Discrete topology

   (ii) Euclidean topology

   (iii) Directed set in respect of a topological space

   (iv) One point compactification

   (v) Disconnected space

   (vi) Cut point

   (vii) Topological group

   (viii) Regular space

   (ix) Locally compact group

   (x) Left Haar measure

Section–B

2. If $X = \{a, b, c\}$ list all topologies which contain exactly four open sets.

Or

Let (X, τ) be a topological space and $C \subset \tau$. Show that C is a basis for τ if and only if for each open set G and for each $x \in G$, there exists an element $B \in C$ s.t. $x \in B \subset G$.

3. Let (X, τ) be topological space, Y be a set and $f: X \to Y$ be a surjective function. Let $\mu = \{V \subset Y : f^{1}(v)$ is open in $X\}$, then show that μ is a topology on Y.

Or

If Y is a subspace of a space X, then show that Y is compact if and only if every cover of Y by sets open in X contains a finite subcover of Y.

4. Let X be a topological space and $Y = (0, 1)$ be a discrete space. Then show that X is disconnected if and only if there exists a continuous surjection map from X onto Y.

Or

Let $f : X \to Y$ be a continuous map from a connected space X onto a topological space Y. Show that Y is connected.

5. Show that the property of a space being regular is hereditary property.

Or

Let H denotes a group that is also a topological space satisfying T_1-axiom. Show that H is a topological group if and only if the map of $H \times H$ into H sending $x \times y$ into $x \cdot y^{-1}$ is continuous.

6. Let G be a topological group. Show that G is locally compact if there is one point of G with a local basis of compact sets.

Or

Let G be a topological group and H a compact subgroup of G. If $\frac{G}{H}$ is compact, then show that G is compact.

Section–C

7. Let X be a topological space and $X = A \cup B$, where A and B are closed sets in X. Let $f : A \to Y$ and $g : B \to Y$ be continuous functions. If $f(x) = g(x)$ $\forall\ x \in A \cap B$, then show that there is a continuous function $h : X \to Y$ such that $h(x) = f(x)\ \forall\ x \in A$ and $h(x) = g(x)\ \forall\ x \in B$.

8. Show that a topological space X is Hausdorff if and only if every set in X converges to atmost one point in X.

9. Show that a non-empty product space is connected if and only if each factor space is connected.

10. Let G be a topological group and H be a subgroup of G. Show that the topological closure of H, i.e. $\bar{H}$, is a subgroup of G.

11. Show that every locally compact group admits a left Haar measure.