AP-2200

M.A./M.Sc. (Final) Examination, 2020

MATHEMATICS

Paper-Opt-IX

(Relativity and Cosmology)

Time allowed : Two hours
Maximum Marks : 100

SECTION – A **(Marks : 2 × 10 = 20)**

Answer all **ten** questions. (Answer limit **50** words) Each question carries **2** marks.

खण्ड – अ **(अंक : 2 × 10 = 20)**

समस्त **दस** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **50** शब्द) । प्रत्येक प्रश्न **2** अंक का है ।

SECTION – B **(Marks : 4 × 5 = 20)**

Answer all **five** questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

खण्ड – ब **(अंक : 4 × 5 = 20)**

समस्त **पाँच** प्रश्नों के उत्तर दीजिए । प्रत्येक प्रश्न में विकल्प का चयन करें (उत्तर सीमा **200** शब्द) । प्रत्येक प्रश्न **4** अंक का है ।

SECTION – C **(Marks : 20 × 3 = 60)**

Answer any **three** questions out of **five.** (Answer limit **500** words) Each question carries **20** marks.

खण्ड – स **(अंक : 20 × 3 = 60)**

पाँच में से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **500** शब्द) । प्रत्येक प्रश्न **20** अंक का है ।

SECTION – A **(Marks : 2 × 10 = 20)**

1. (i) Define proper time. **2**

   (ii) A particle with a mean proper life of 1μ second moves through the laboratory at 2.7×10^{10} cm./sec. Find the distance transversed without taking relativity into account. **2**

(iii) Write Minkowski's equation of motion.

(iv) Define world points and world lines.

(v) Write the statement of principle of general covariance.

(vi) Write the statement according to Einstein the principle of equivalence.

(vii) What are crucial tests of general relativity ?

(viii) Find the trace of Einstein-tensor $G_i^j \frac{-1}{2} R\, g_i^j$ in space time.

(ix) Write Einstein line element for static homogeneous universe.

(x) Define cosmological models.

SECTION – B

(Marks : 4 × 5 = 20)

2. Prove that simultaneity has only a relative and not an absolute meaning.

OR

Prove that four dimensional volume element dx dy dz dt is invariant under Lorentz transformations.

3. Derive Einstein relation $E^2 = c^2 p^2 + m_0^2 c^4$ where the symbols have their usual significance.

OR

A red hot sphere of iron (specific heat capacity = 0.11 cal/gm °C) weights to 1 kg. If the sphere cools 1200 °C, what is the equivalent loss in mass ?

4. Explain the statement that the mass of the sun which is 1.99×10^{33} gms, becomes in gravitational units 1.47 kilometres.

OR

Show that the divergence of the energy tensor vanishes and in the usual notation prove that $G = 8\pi T$.

Write analogues of Kepler's laws. **4**

OR

By what angle would a light grazing the sun bend ? Mass of the sun = 2×10^{33} g. Radius of the sun = 6.96×10^{10} cm, Gravitational Constant = 6.67×10^{-8} cm^3/g.

6. Discuss motion of a test particle in the Einstein universe. **4**

OR

Discuss comparison of De-Sitter model with actual universe.

SECTION – C (Marks : 20 × 3 = 60)

7. Show that the results of two successive Lorentz transformations is a Lorentz transformation. **20**

8. Formulate energy momentum vector in the space time of special relativity and show that it is Lorentz invariant. Prove that $p^2 - \frac{E^2}{C^2} = -m_0^2c^2$. **20**

9. Obtain the general relativistic equation of motion of a particle in gravitational field through a principle of least action. **20**

10. Discuss the structure of the world given by the equation $R_i^j - \frac{1}{2}\ \delta_i^j\ (R - 2\lambda) = -8\pi\ T_i^j$, where λ is constant. Verify that for empty space $R = 4\lambda$ and $T_i^j = 0$. **20**

11. Discuss the transformation properties of the magnetic and the electric fields. **20**
