# AP-2192

## M.A./M.Sc. (Final) Examination, 2020

## MATHEMATICS

### Paper-Opt-III
### (Mechanics)

*Time allowed : Two hours*
*Maximum Marks : 100*

**SECTION – A** **(Marks : 2 × 10 = 20)**

Answer all **ten** questions (Answer limit **50** words). Each question carries **2** marks.

**खण्ड – अ** **(अंक : 2 × 10 = 20)**

समस्त **दस** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **50** शब्द) । प्रत्येक प्रश्न **2** अंक का है ।

**SECTION – B** **(Marks : 4 × 5 = 20)**

Answer all **five** questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

**खण्ड – ब** **(अंक : 4 × 5 = 20)**

समस्त **पाँच** प्रश्नों के उत्तर दीजिए । प्रत्येक प्रश्न में विकल्प का चयन करें (उत्तर सीमा **200** शब्द) । प्रत्येक प्रश्न **4** अंक का है ।

**SECTION – C** **(Marks : 20 × 3 = 60)**

Answer any **three** questions out of **five** (Answer limit **500** words). Each question carries 20 marks.

**खण्ड – स** **(अंक : 20 × 3 = 60)**

**पाँच** में से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **500** शब्द) । प्रत्येक प्रश्न **20** अंक का है ।

### SECTION – A

1. Attempt all **ten** questions. (Answer limit **50** words)

   (i) Write the property that momental ellipsoid possesses.

   (ii) Define the centre of percussion.

   (iii) Define the degree of freedom.

   (iv) Write the Lagrange's Equations in Generalized co-ordinates.

   (v) Write the Hamilton-Jacobi equation.

(vi) Write the Jacobi's identity.

(vii) Define partial differential equation.

(viii) Define non-homogeneous equation.

(ix) Explain mean value formula.

(x) Write the non-homogeneous wave equation.

## SECTION – B

2. Find the product of Inertia of an elliptic quadrantal disc with respect to its axes.

**OR**

To deduce the general equation of motion of a rigid body from D'alembert's Principle.

3. Find the equation of motion in two dimensions under finite forces.

**OR**

If a horizontal cylinder of radius a rolling inside a perfectly rough hollow horizontal cylinder of radius b (> a). Then classify each of the following dynamical systems.

(i) Scleronomic or Rheonomic.

(ii) Holonomic or Non-holonomic.

(iii) Conservative or non-conservative.

4. Obtain the Hamilton's canonical equations.

**OR**

If $u = u(q_r, p_r, t)$ and $H(p_r, q_r, t)$ is the Hamiltonian, then show that $\frac{dH}{dt} = [u, H] + \frac{\partial u}{\partial t}$.

5. Solve : $z^2(p^2x^2 + q^2) = 1$.

**OR**

Solve the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$

Given $u(x, 0) = \sigma e^{-3x}$.

6. Solve : $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $t > 0, 0 \le x \le 1$

BCs : $u(0, t) = 2, u(1, t) = 3$

and $u(x, 0) = x(1 - x)$

**OR**

Solve : $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

BCs : (i) $y = 0$ when $x = 0$

(ii) $y = 0$ when $x = l$

(iii) $\frac{\partial y}{\partial t} = 0$ when $t = 0$

(iv) $y = \begin{cases} \frac{2k}{l} x, & 0 < x < \frac{l}{2} \\ \frac{2k}{l}(l - x), & \frac{l}{2} < x < l \end{cases}$, when $t = 0$.

## SECTION – C

Attempt any **three** questions out of **five**.

7. A bent lever, whose arms are of length a and b, the angle between them being $\alpha$, makes small oscillations in its own plane about the fulcrum; prove that the length of the corresponding simple pendulum is

$$\frac{2}{3} \frac{a^3 + b^3}{\sqrt{a^4 + 2a^2b^2\cos\alpha + b^4}}.$$

8. A heavy ring, of radius a, is moving in its own plane which is vertical. At a certain instant when its velocity is V horizontally from left to right and the angular velocity is $\left(\frac{V}{2a}\right)$ clockwise, the highest point of ring is suddenly fixed. Prove that the ring will describe a complete revolution about the point of fixing, if $V^2 \ge 32\,ag$.

9. If the transformation equations between two sets of co-ordinates are

$$P = 2(1 + q^{1/2}\cos p)\, q^{1/2}\sin p,\ Q = \log(1 + q^{1/2}\cos p),$$

then show that

(i) the transformation is canonical.

(ii) the function $G_3$ which generates this transformation is $G_3 = -(e^Q - 1)^2 \tan p$.

10. Solve : (a) $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y} + \sin(2x + y)$.

(b) $(p^2 + q^2)\, y = qz$.

11. Solve the three dimensional wave equation :

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial u}{\partial \theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial \phi^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$