ASP-2221

M.A./M.Sc. (Final) Examination, 2019

MATHEMATICS

Paper – Opt - V

(Differential Geometry of manifolds)

Time allowed : Three hours

*Maximum Marks : **100***

SECTION – A **(Marks $2 \times 10 = 20$)**

Answer all **ten** questions (Answer limit **50** words). Each question carries **02** marks.

खण्ड – अ **(अंक $2 \times 10 = 20$)**

समस्त **दस** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **50** शब्द) । प्रत्येक प्रश्न **02** अंक का है ।

SECTION – B **(Marks $4 \times 5 = 20$)**

Answer all **five** questions. Each question has internal choice (Answer limit **200** words). Each question carries **04** marks.

खण्ड – ब **(अंक $4 \times 5 = 20$)**

समस्त **पाँच** प्रश्नों के उत्तर दीजिए । प्रत्येक प्रश्न में विकल्प का चयन करें (उत्तर सीमा **200** शब्द) । प्रत्येक प्रश्न **04** अंक का है ।

SECTION – C **(Marks $20 \times 3 = 60$)**

Answer any **three** questions out of **five** (Answer limit **500** words). Each question carries **20** marks.

खण्ड – स **(अंक $20 \times 3 = 60$)**

पाँच में से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **500** शब्द) । प्रत्येक प्रश्न **20** अंक का है ।

SECTION – A **(Marks $2 \times 10 = 20$)**

1. (i) Define Jacobian map. 2

   (ii) Define Exterior derivative. 2

   (iii) Define lie groups and lie Algebras. 2

   (iv) Define lie Transformations. 2

   (v) Define Associated fibre bundle. 2

(vi) Define Riemannian Connection. 2

(vii) Define Sectional Curvature. 2

(viii) Define conformal curvature tensor. 2

(ix) Define Narmal's Ganss formulae. 2

(x) Define F-connection. 2

SECTION – B

(Marks $4 \times 5 = 20$)

2. Explain one Parameter group of transformations. 4

OR

Explain immersion and Embendings. $2 + 2 = 4$

3. Show that lie bracket is biliner. 4

OR

Explain lie groups homomorphism and isomorphism. $2 + 2 = 4$

4. Explain induced bundle and tangent bundle. $2 + 2 = 4$

OR

Explain Riemannian mainfolds. 4

5. If a mainfold Vn $n > 3$ is conformly flat, then to prove : 4

$$(D_\lambda L)(\mu, v) = (D_\mu L)(\lambda, v)$$

OR

Explain projective curvature Tensor.

6. Show that D is a Riemannian connection in V_n and V is vector valued in V_m is symmetric.

OR

Explain contravariant and covariant almost analytic vector fields : $2 + 2 =$

SECTION – C (Marks $20 \times 3 = 60$)

7. (a) Show that the n-dimension sphere S^n is a differentiable manifold (on a smooth manifold) **10**

   (b) Let $V_{\lambda p}$ be the set of all p-forms. Then L_λ is a mapping $L_\lambda : V_{\wedge p} \to V_{\wedge p}$, then to prove that lie derivative preserve forms. **10**

8. (a) The general linear group G2 (n, R) is a differential main folds of dimension n^2. **10**

   (b) If it is an Abstract subgroup and a regular sub manifold of a lie group G. Then to prove it is a Lie subgroup of G. **10**

9. Let X, Y be mainfolds and $f_0 : X \to Y$ a morphism. Let α, β be vector bundles over X, Y respectively and Let f, g, $\alpha \to \beta$ be two vector bundle morphism over f_0 then the map f + g defined by the formula $(f + g)_x = f_x + g_x$ is also a vector bundle morphism.

   Further more if $\psi : Y \to R$ is a function on Y, then the map ψ defined by

   $$(\psi f)_x = \psi\,(f(x))f_x$$

   is also a vector bundle morphism. **10 + 10 = 20**

10. (a) Show that the conformal curvature tensor vanishes for Mainfolds with constant Riemannian curvature. **10**

    (b) Show that the only mainfold whose geodesics correspond to the geodesics of a manifold of constant Riemannian curvature are mainfolds of constant Riemannian- curvature. **10**

11. (a) State and prove Weingraten equation. **2 + 8 = 10**

    (b) State and prove generalized Gauss and Mainardi Codozzi equation. **2 + 4 + 4 = 10**