AP-2191

M.A./M.Sc. (Final) Examination, 2020

MATHEMATICS

Paper-Opt-II
(Advance Discrete Mathematics)

Time allowed : Two hours
Maximum Marks : 100

SECTION – A **(Marks : 2 × 10 = 20)**

Answer all **ten** questions (Answer limit **50** words). Each question carries **2** marks.

खण्ड – अ **(अंक : 2 × 10 = 20)**

समस्त **दस** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **50** शब्द) । प्रत्येक प्रश्न **2** अंक का है ।

SECTION – B **(Marks : 4 × 5 = 20)**

Answer all **five** questions. Each question has internal choice (Answer limit **200** words). Each question carries **4** marks.

खण्ड – ब **(अंक : 4 × 5 = 20)**

सभी **पाँच** प्रश्नों के उत्तर दीजिए । प्रत्येक प्रश्न में आंतरिक विकल्प का चयन करें (उत्तर सीमा **200** शब्द) । प्रत्येक प्रश्न **4** अंक का है ।

SECTION – C **(Marks : 20 × 3 = 60)**

Answer any **three** questions out of **five** (Answer limit **500** words). Each question carries **20** marks.

खण्ड – स **(अंक : 20 × 3 = 60)**

पाँच में से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए (उत्तर सीमा **500** शब्द) । प्रत्येक प्रश्न **20** अंक का है ।

SECTION – A

1. (i) Define Monoids.

   (ii) State Basic Homomorphism theorem.

   (iii) Define Atoms.

   (iv) Define Distributive Lattices.

   (v) Define Planner graph.

(vi) State Kuratowski's theorem.

(vii) Define spanning tree.

(viii) Define Centre of tree.

(ix) Define Finite State Machines.

(x) State Kleen's theorem.

SECTION – B

2. Prove that validity of the Argument.

$$\frac{P \quad p \to q}{\therefore q}$$, i.e. $P \wedge (P \to q) \to q$ is a tautology.

OR

Let A be the set of people of different heights. Define binary operation $\otimes$ by,

$a \otimes b$ = Taller of a and b,

then $\{a, \otimes\}$ is a monoid.

3. Let $(L, \leq)$ be a bounded lattice with least element 0 and greatest element 1, then for any element $a \in L$, show that

(i) $a \vee 1 = 1$ and $a \wedge 1 = a$

(ii) $a \vee 0 = a$ and $a \wedge 0 = 0$

OR

To prove that $x' + y'$ is the complement of xy, we have to prove

(i) $xy + (x' + y') = 1$

(ii) $(xy)(x' + y') = 0$

4. Show that the graph $k_{3,3}$ is non-planar.

OR

Derive Adjacency and Incidence Matrix for directed and undirected graph with examples.

Show that in a binary tree number of pendent vertices is $\frac{(n+1)}{2}$.

OR

A tree has 1 vertex of degree 1, 2 vertices of degree 2, 3 vertices of degree 3, and n vertices of degree n, then prove that no such tree exists.

6. Find the Language generated by grammar

G = (V, T, P, S), where V = {S, a, b}, T = {a, b} and

P = {S $\nrightarrow$ a as S $\rightarrow$ a, S $\rightarrow$ b}.

OR

Write a note of nondeterministic finite state automation.

SECTION – C

7. (i) State and proof Law of Syllogism.

(ii) Prove that the set of all n^{th} roots of unity is a group with respect to multiplication.

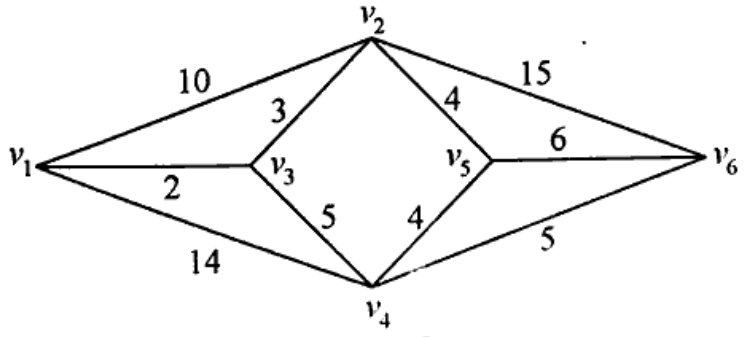
8. (i) If (L, $\leq$) is a distributive lattice, then for any a, b, c $\in$ L, show that

$$a \vee b = a \vee c \text{ and } a \wedge b = a \wedge c \Rightarrow b = c.$$

(ii) Simplify the Boolean expression $E(x, y) = xy + x'y + x'y'$ by drawing k-Map.

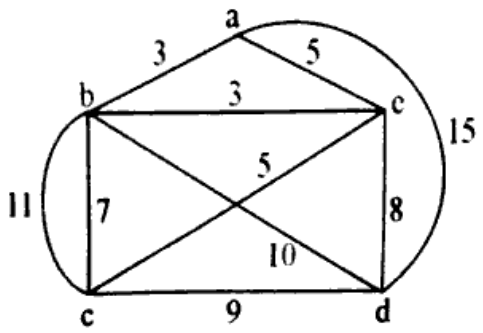
9. (i) A simple graph (i.e. a graph without parallel edges and self loops) with n vertices and k components can have at most (n – k) (n - k + 1)/2 edges.

(ii) Apply Dijkstra's algorithm to find the shortest path from v_1 to v_6 in the weighted graph.



10. (i) In any connected graph of n vertices and e edges every spanning tree has $(n - 1)$ tree branches and $e - n + 1$ chords.

(ii) Use Krushkal's algorithm to find the minimum time (in hour) for travelling five cities whose road map is shown in the following graph :



11. (i) Let $M = \{A, S, Y, f, \$_0\}$ be a non-deterministic finite state automation, let

(a) $S' = P(S)$

(b) $A' = A$

(c) $\$_0^1 = \{\$_0\}$

(d) $Y' = \{X \subseteq S : X \cap Y \neq \phi\}$

(e) $f'(X, x) = \phi$ if $X = \phi$ $\bigcup_{\$ \in X} f(\$, x)$: if $X \neq \phi$

then show that the finite sate automation.

$M' = \{A', S', Y', f', \$_0'\}$ is equivalent to M.

(ii) State and proof pumping Lemma.
